Doing Things With Derivatives

Math 130 - Essentials of Calculus

3 March 2021

Tangent Lines

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2. $f(x) = x + \sqrt{x}$ *at* $(4, 6)$

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Use linear approximation to approximate the value of the given number:

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The Cost Function

A cost function encodes the production costs a company incurs as a function of units produced. This includes things such as *fixed costs* (such as rent or costs of machines), but also *variable costs* such as the cost of raw materials, employee wages, and the like. The *cost function* is the sum of all these various costs, we will call $C(q)$ which gives the total cost that a company incurs in producing q units of a particular good. We expect the cost function to be an increasing function.

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$$C(q) = a + bq + cq^2$$

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a represents the fixed costs here.

Average and Marginal Cost

Definition (Average Cost)

If $C(q)$ is the total cost of producing q units of a good or service, then the average cost per unit is

$$\frac{C(q)}{q}$$

If only a few units are produced, the average cost will be high since the fixed costs are distributed among few items, however if the number of units produced is increased, it can be expected that the average cost should decrease. If the average cost decreases with the production of more units, it's possible that the product could be sold at a lower price.

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Definition (Marginal Cost)

If $C(q)$ is the total cost of producing q units of a good or service, then the marginal cost per unit is

$$C'(q) = \frac{dC}{dq}$$

which is the instantaneous rate of change of cost with respect to the number of units produced.

Example

Suppose a company has estimated that the cost, in dollars, of producing q items per week is $C(q) = 3000 + 13q - 0.01q^2 + 0.000003q^3$.

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3. Find the marginal cost function. What is the marginal cost when 1500 units are produced?

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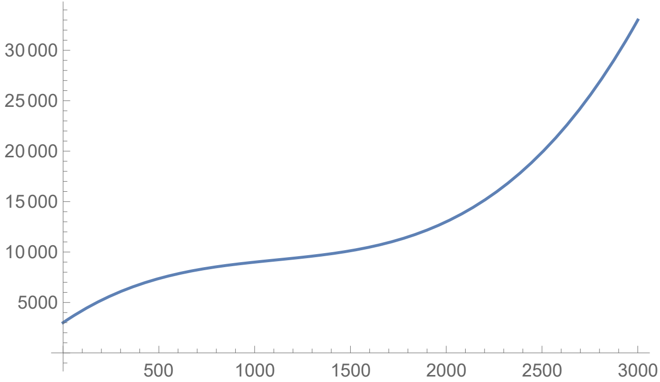
3. Find the marginal cost function. What is the marginal cost when 1500 units are produced?
4. What is the actual cost of the 1501*st* item?

Minimizing Average Cost

Below is the graph of the cost function from the last example



We can see here that for lower production levels, the cost increases, but at a rate which is decreasing, so that marginal costs are decreasing. However, eventually an inflection point is reached where the marginal cost starts increasing again.

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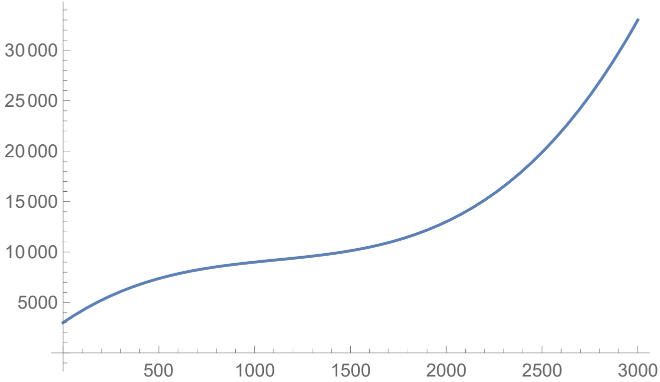

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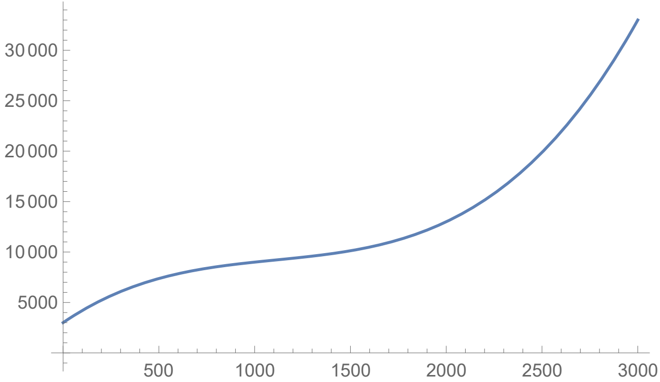
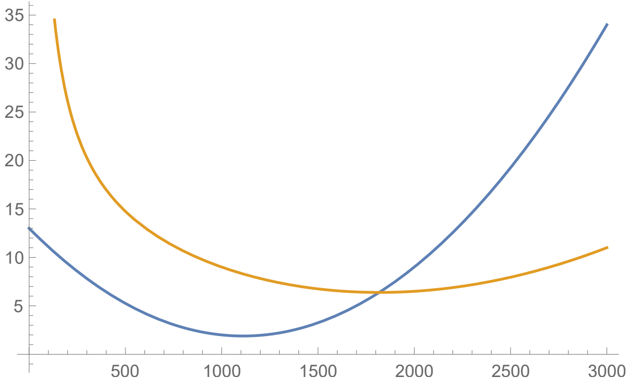


We can see here that for lower production levels, the cost increases, but at a rate which is decreasing, so that marginal costs are decreasing. However, eventually an inflection point is reached where the marginal cost starts increasing again. If marginal cost is increasing, does it make sense to increase production? As long as the marginal cost is less than the average cost, then yes.

Minimizing Average Cost

Below is the graph of the marginal (in blue) and average (in orange) cost function from the last example



It makes sense because if the marginal cost is less than the average cost, producing an additional unit will lower the average cost. Conversely, if the marginal cost becomes higher than the average cost, producing an additional item will increase the average cost.

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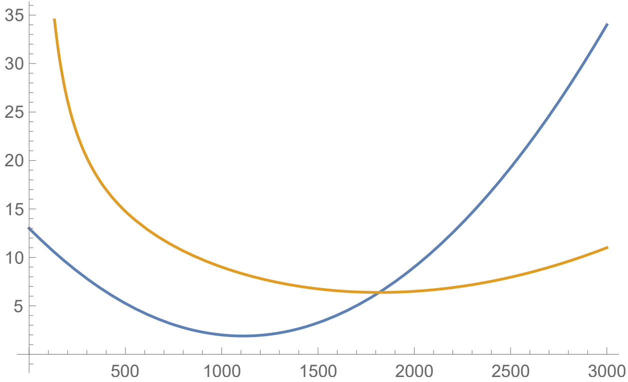

It makes sense because if the marginal cost is less than the average cost, producing an additional unit will lower the average cost. Conversely, if the marginal cost becomes higher than the average cost, producing an additional item will increase the average cost. What can this tell us about minimizing the average cost?

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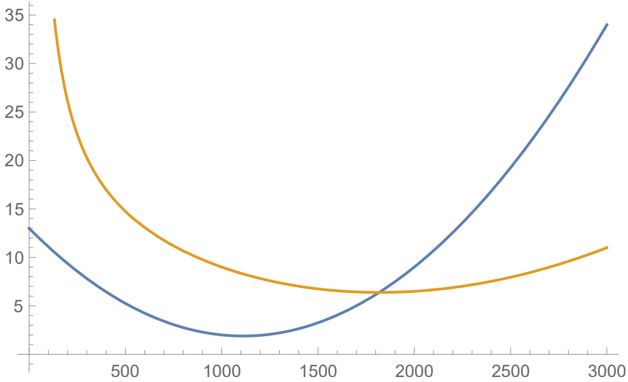

Typically, *the minimum average cost per item occurs when the marginal cost is the same as average cost*, as the previous graph will show.

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A small furniture manufacturer estimates that the cost, in dollars of producing q units of a particular chair each month is given by

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How many chairs should be produced in order to minimize the average cost of each chair?

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A baker estimates that it costs

$$C(q) = 0.01q^2 + 2q + 250$$

dollars each day to bake q loaves of bread. How many loaves should be baked daily in order to minimize the average cost?